Minimizing Logic Circuits

<u>GOAL</u>: Find the minimal realization of the function

A	*B*	*C*	*f*(*A*,*B*,*C*)
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1 } $AB\overline{C}$
1	1	1	1 } ABC

Minimizing Logic Circuits (cont'd)

Algebraic Solution:

Write a canonical sum-of-products expression

$$f(A,B,C) = AB\overline{C} + ABC$$

Apply distributivity **OR**

$$f(A,B,C) = AB(\overline{C} + C)$$

Apply $\overline{C} + C = 1$ **and** $AB1 = AB$

$$f(A,B,C) = AB$$

Needs one 2-input AND gate.

Minimizing Logic Circuits (cont'd)

<u>GOAL</u>: Find the AND-OR two-level minimal realization of the function (Find the minimum SOP.)

A	*B*	*C*	*f*(*A*,*B*,*C*)
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

Reflected Gray Code

C \ *AB*	00	01	11	10
0	0	0	1	0
1	0	0	1	0

AB

Karnaugh Map

Minimizing Logic Circuits (cont'd)

Karnaugh Map Solution:

Circle the two adjacent pair of 1's and write the corresponding expression

$$f(A,B,C) = AB$$

Minimizing Logic Circuits (cont'd)

Karnaugh Maps were developed by Maurice Karnaugh, a Bell Laboratories engineer in 1953 and presented as

Maurice Karnaugh, "The map method for synthesis of combinational logic circuits," ***Transactions of the American Institute of Electrical Engineers*****, 72, 1, 593-599, November, 1953**

Minimizing Logic Circuits (cont'd)

GOAL: Find the AND-OR two-level minimal realization of the function

A	*B*	*C*	*f*(*A*,*B*,*C*)	
0	0	0	0	
0	0	1	0	
0	1	0	0	
0	1	1	0	
1	0	0	0	
1	0	1	1	$A\bar{B}C$
1	1	0	1	$AB\bar{C}$
1	1	1	1	ABC

Minimizing Logic Circuits (cont'd)

Algebraic Solution:

Write a canonical sum-of-products expression

$$f\left(A,B,C\right)=A\overline{B}C+ABC+AB\overline{C}$$

Apply $ABC=ABC+ABC$

$$f\left(A,B,C\right)=A\overline{B}C+ABC+ABC+AB\overline{C}$$

Apply distributivity

$$f\left(A,B,C\right)=\left(B+\overline{B}\right)AC+AB\left(C+\overline{C}\right)$$

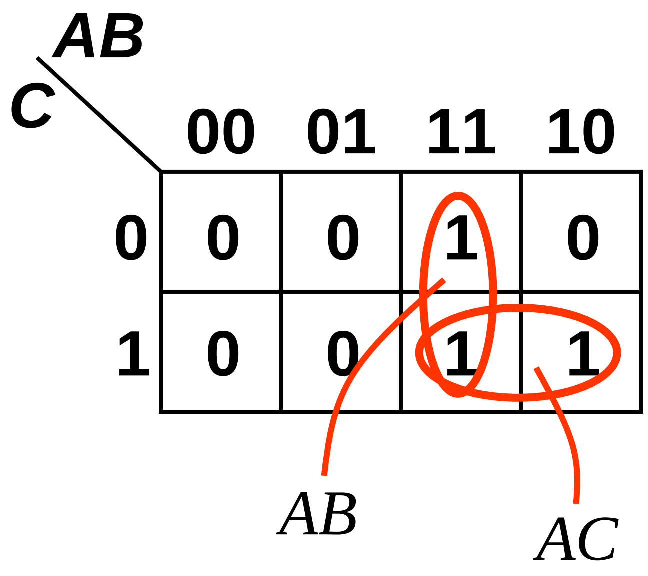
Apply $B+\overline{B}=1$ **and** $A1C=AC$

$$f\left(A,B,C\right)=AC+AB$$

Minimizing Logic Circuits (cont'd)

<u>GOAL</u>: Find the AND-OR two-level minimal realization of the function

A	*B*	*C*	*f*(*A*,*B*,*C*)
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1



Minimizing Logic Circuits (cont'd)

Karnaugh Map Solution:

Circle the two adjacent pairs of 1's and write the corresponding expression

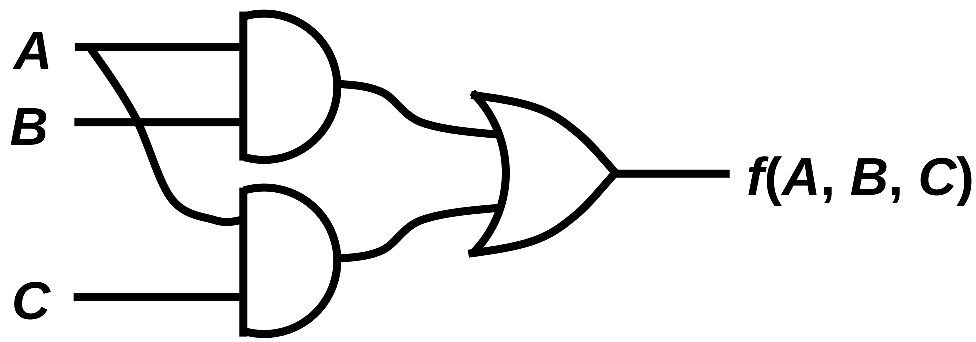
$$f\left(A,B,C\right)=AB+AC$$

Minimizing Logic Circuits (cont'd)

Minimal AND-OR two-level circuits are not necessarily minimal. Consider

$$f\left(A,B,C\right)=AB+AC$$

which can be realized as

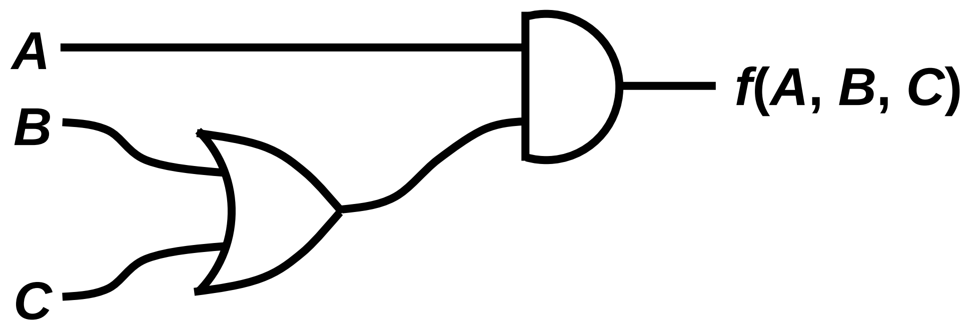


Minimizing Logic Circuits (cont'd)

However, we can write

$$f\left(A,B,C\right)=AB+AC=A\left(B+C\right)$$

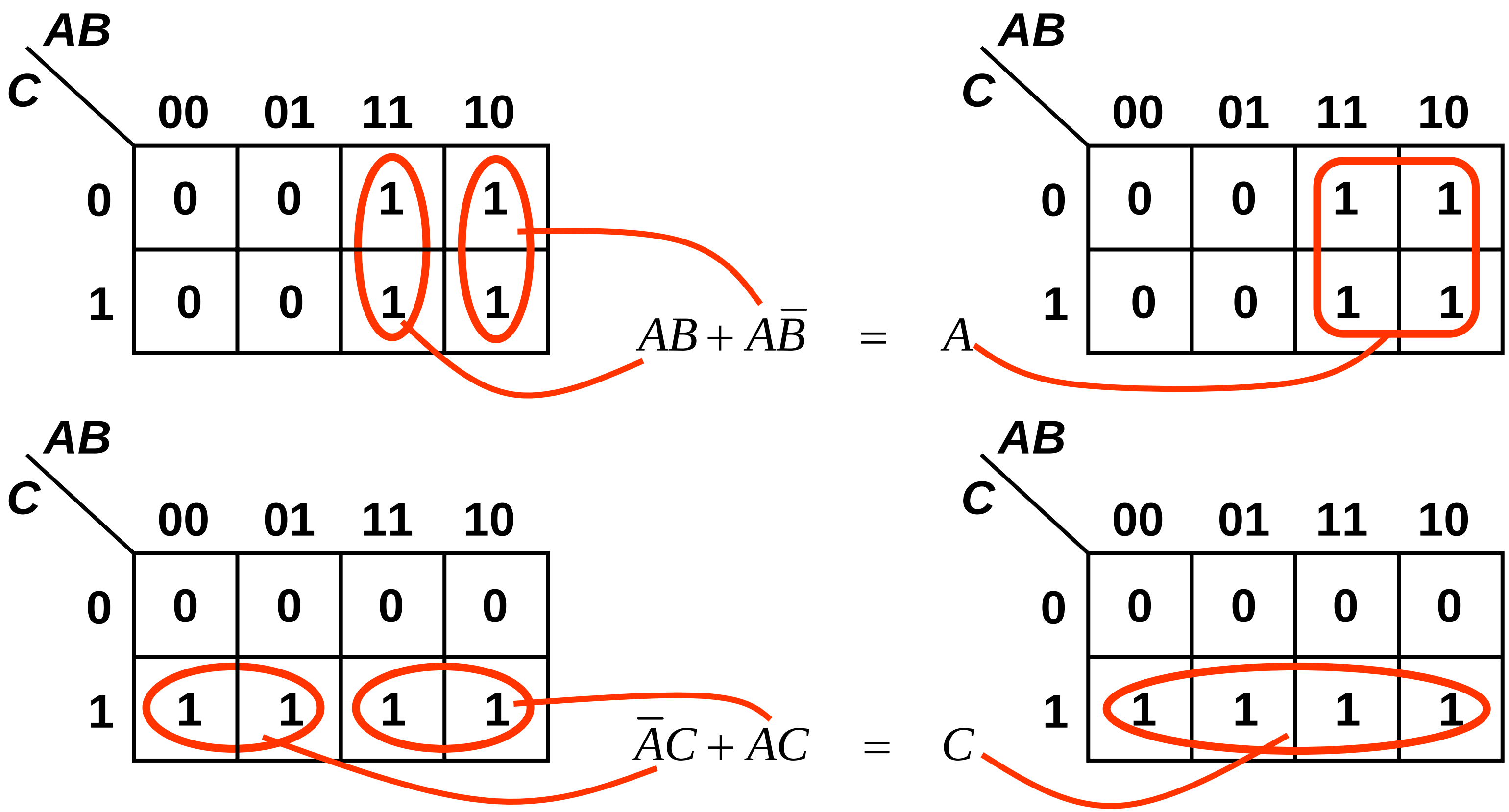
which can be realized as



This is NOT an AND-OR two-level circuit. Rather, it is an OR-AND two-level circuit.

Other Combinations

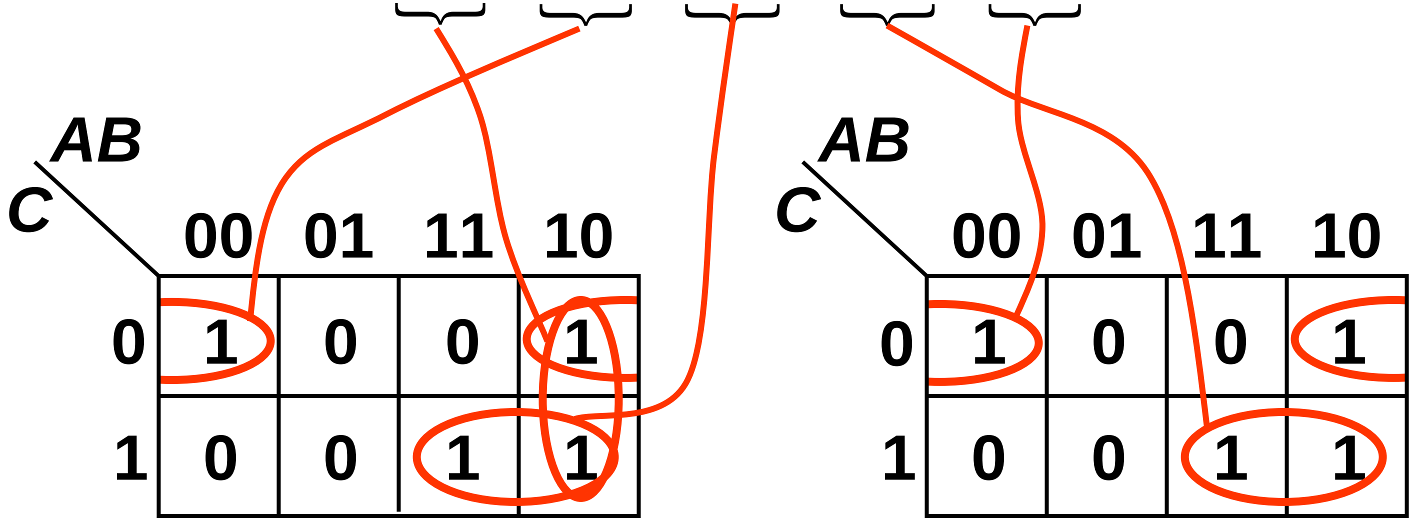
From previous slides, a pair of 1's yields a single product term. However, other combinations are possible.



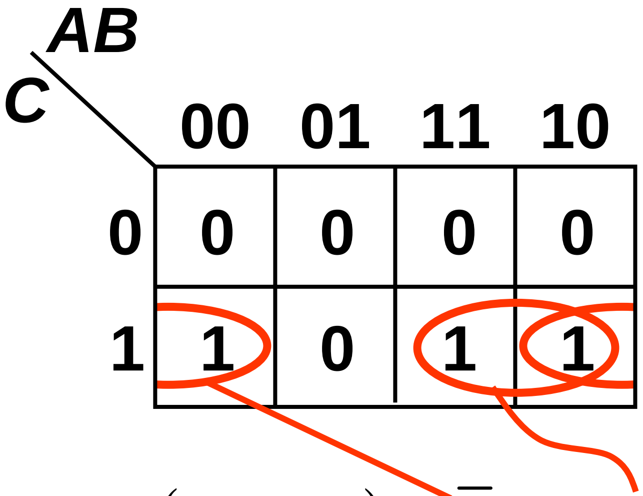
A “Look-See” Proof of Consensus

Use the Karnaugh Map to prove a result stated previously. This is called “consensus”.

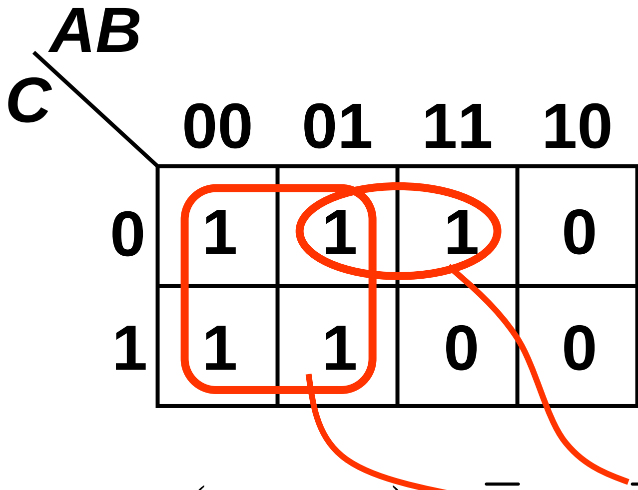
$$\underbrace{A\overline{B}} + \underbrace{\overline{B}\,\overline{C}} + \underbrace{AC} = \underbrace{AC} + \underbrace{\overline{B}\,\overline{C}}$$



Other Examples

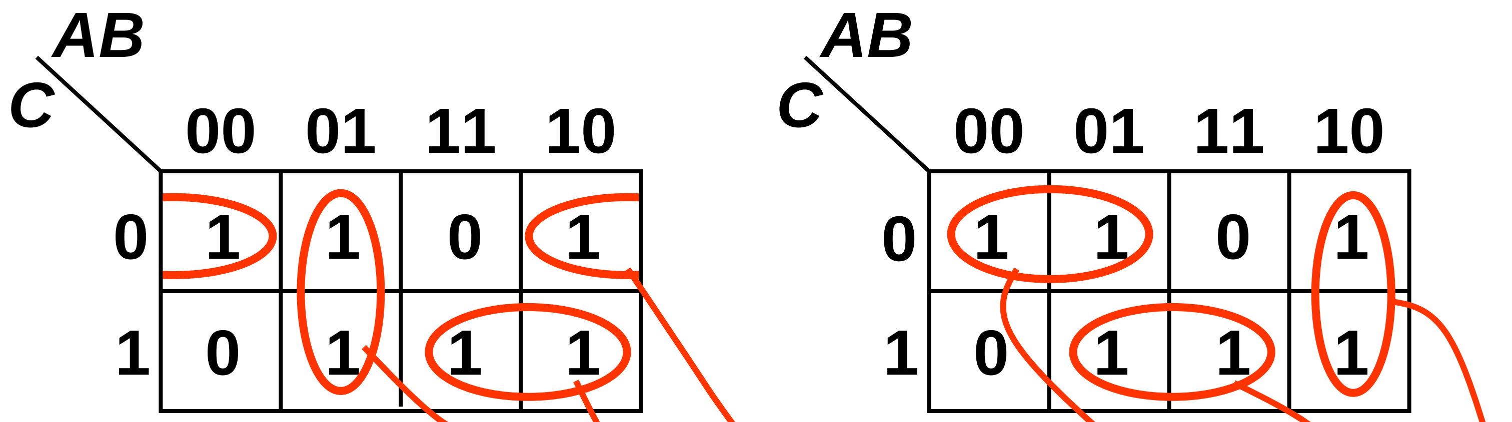


$$f\left(A,B,C\right)=\overline{B}C+AC$$



$$f\left(A,B,C\right)=\overline{A}+B\overline{C}$$

Other Examples (cont'd)



$$f\left(A,B,C\right)=\overline{B}\,\overline{C}+\overline{A}B+AC \qquad f\left(A,B,C\right)=\overline{A}\,\overline{C}+BC+A\overline{B}$$

Procedure for Karnaugh Map Circling

1. **Start by covering single 1 cells that cannot combine with any other 1 cell. Circle 1 cells that can combine in only one way with one other 1 cell. Continue: circle 1's that combine uniquely in a group of 4, 8, 16, etc.**
2. **A minimal expression is obtained as a collection of 1's that are as large as possible and as few as possible, so that every 1 cell is covered.**

Four-Variable Karnaugh Map

CD \ AB	00	01	11	10
00	0	0	1	1
01	1	0	0	0
11	0	1	1	0
10	1	1	1	1

$$f(A,B,C,D) = \bar{A}\bar{B}\bar{C}D + A\bar{D} + BC + C\bar{D}$$

CD \ AB	00	01	11	10
00	0	0	1	0
01	1	1	1	0
11	0	1	1	1
10	0	1	0	0

$$f(A,B,C,D) = AB\bar{C} + \bar{A}\bar{C}D + \bar{A}BC + ACD$$

Four-Variable Karnaugh Map (cont'd)

CD \ AB	00	01	11	10
00	0	0	1	1
01	0	1	1	0
11	0	0	1	1
10	0	1	1	0

$$f\left(A,B,C,D\right) = A\overline{C}\overline{D} + B\overline{C}D + ACD + BC\overline{D}$$

Forbidden Circlings

CD \ AB	00	01	11	10
00	0	1	1	1
01	0	0	0	0
11	1	0	0	0
10	0	1	0	0

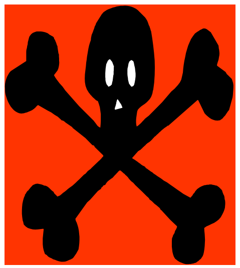
$$\bar{A}B\bar{C}\bar{D} + AB\bar{C}\bar{D} + A\bar{B}\bar{C}\bar{D}$$

$$B\bar{C}\bar{D} + A\bar{C}\bar{D}$$

Not combinable

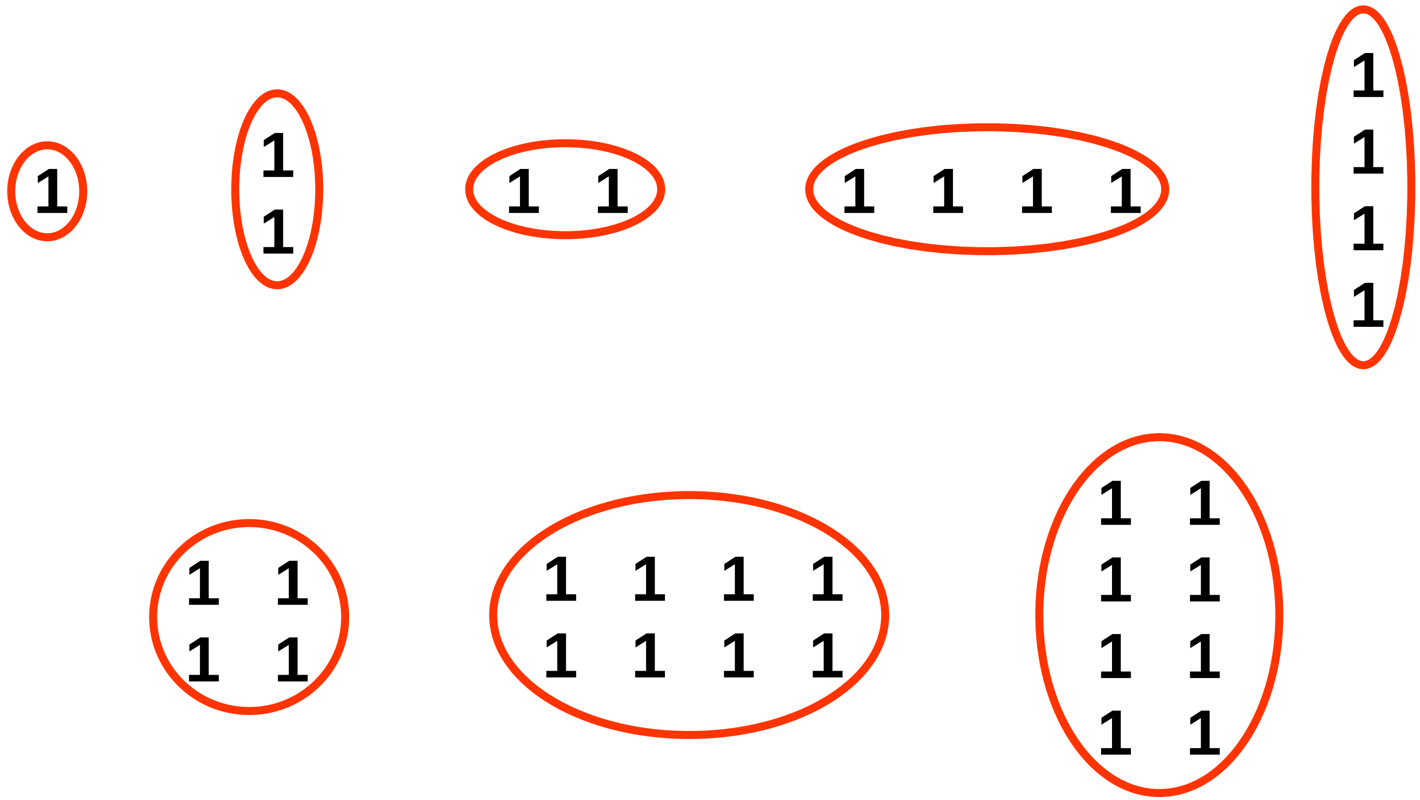
$$\bar{A}\bar{B}CD + \bar{A}BC\bar{D}$$

Not combinable



Warning !

Acceptable Circlings



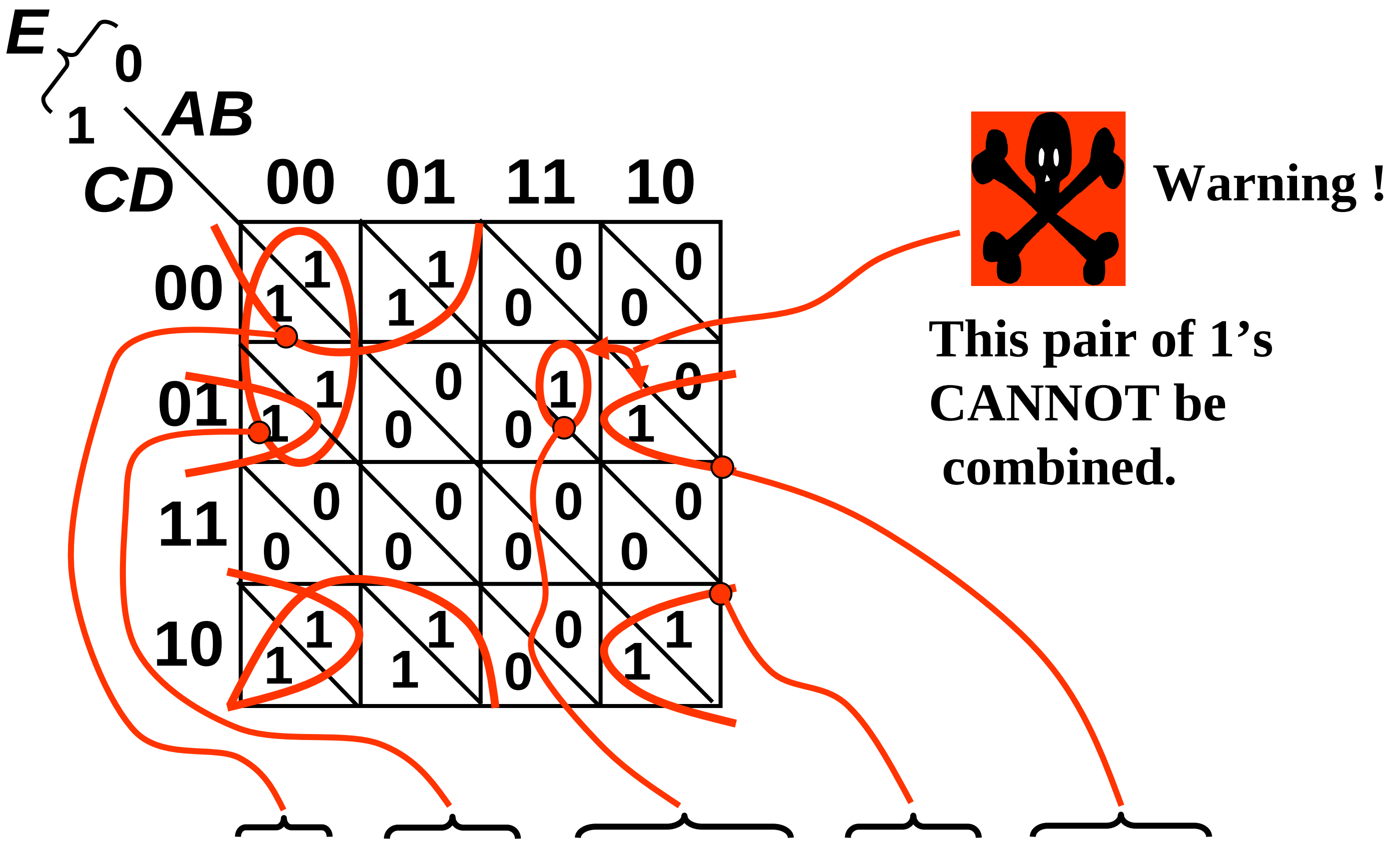
Five-Variable Karnaugh Map

Reflected Gray Code

DE \ ABC	000	001	011	010	110	111	101	100
00	1	0	0	0	0	0	0	0
01	0	1	0	0	0	0	1	0
11	0	0	0	0	0	0	0	0
10	1	0	0	1	1	1	1	0

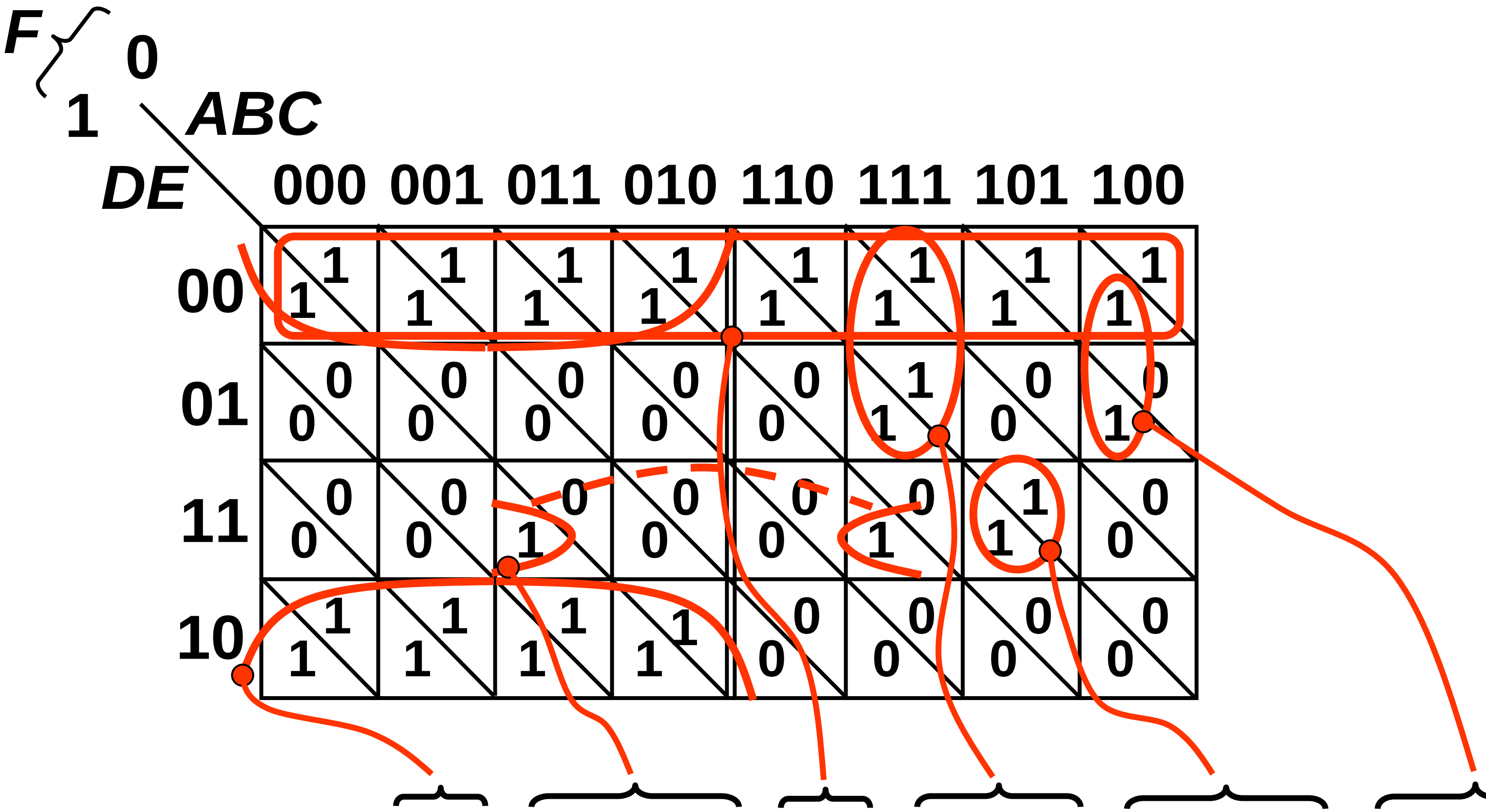
$$f\left(A,B,C,D,E\right)=\overline{A}\,\overline{B}\,\overline{C}\,\overline{E}+\overline{B}C\overline{D}E+B\overline{C}D\overline{E}+ACD\overline{E}$$

Five-Variable Karnaugh Map



$$f(A,B,C,D,E) = \bar{A}\bar{D} + \bar{A}\bar{B}\bar{C} + AB\bar{C}D\bar{E} + \bar{B}C\bar{D} + \bar{B}\bar{C}DE$$

Six-Variable Karnaugh Map



$$f\left(A,B,C,D,E,F\right)=\bar{A}\bar{E}+BCDEF+\bar{D}\bar{E}+ABC\bar{D}+A\bar{B}CDE+A\bar{B}\bar{C}\bar{D}F$$